

Manifestation of nonclassical Berry phase of an electromagnetic field in atomic Ramsey interference

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The Berry phase acquired by an electromagnetic field undergoing an adiabatic and cyclic evolution in phase space is a purely quantum-mechanical effect of the field. However, this phase is usually accompanied by a dynamical contribution and can not be manifested in any light-beam interference experiment because it is independent of the field state. We here show that such a phase can be produced using an atom coupled to a quantized field and driven by a slowly changing classical field, and it is manifested in the atomic Ramsey interference oscillations. We also show how this effect may be applied to one-step implementation of multi-qubit geometric phase gates, which is impossible by previous geometric methods. The effects of dissipation and fluctuations in the parameters of the pump field on the Berry phase and visibility of the Ramsey interference fringes are analyzed.

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I. INTRODUCTION

When the Hamiltonian of a quantum system, depending on a set of parameters, is adiabatically changed along a closed curve in parameter space, then the quantum system in an eigenstate of the Hamiltonian will acquire a purely geometric phase in addition to the usual dynamical phase [1]. Compared with the dynamic phase, the Berry phase is given by a circuit integral in parameter space and is independent of energy and time. During the past few decades, Berry phase has been the subject of a variety of theoretical and experimental investigations. Besides the fundamental interest, Berry phase has many important applications, ranging from optics and molecular physics to quantum computation by geometric means [2-6]. Since the Berry phase only depends upon the geometry of the evolution path, it is robust against fluctuation perturbations that affect the dynamical phase [7]. This feature makes quantum logic gates based on geometric phases have potential fault-tolerance in the presence of noise perturbation. The geometric phase has been generalized to the case of nonadiabatic, noncyclic, and nonunitary evolution of a quantum system [8,9].

The Berry phase and its robustness against noise perturbations has been experimentally tested in various two-state systems [10-14]. Optical experiments have been performed to observe Berry phase of light beams [15-17] that can be understood as a classical effect following the Maxwell equations [18]. The observation of this effect at the single-photon level has also been reported [19], but the Berry phase without classical origin has not been directly measured for any quantum harmonic oscillator in continuous-variable (infinite-dimensional) states. The cyclic and adiabatic displacement in phase space is the simplest quantum-mechanical transformation that can produce a nonclassical Berry phase for a continuous-variable system. Unfortunately, no scheme has been proposed for realizing such transformation in a realistic phys-

ical system without introducing the dynamical phase. Furthermore, the Berry phase acquired through such a transformation cannot be manifested in any optical interference experiment. This is due to the fact that the interference of light fields is fundamentally different from that of particles. It is the relative phase of states that manifests in the latter case, while it is the relative phase of the electric amplitudes in the former case. Thus, the geometric phase measured in a light-beam interference experiment is the Hannay angle rather than the Berry phase [20]. For a cyclic displacement evolution, the acquired Berry phase is independent of the field state and the Hannay angle is zero.

Here we show that the Berry phase of a quantized field, associated with a cyclic and adiabatic displacement in phase space, can be produced and measured with an atomic quantum bit (qubit) that is coupled to the quantized field and driven by a classical pump field. By means of variation of the parameters of the pump field the quantized field undergoes an adiabatic and cyclic evolution in phase space, conditional upon the state of the qubit. The two qubit states correspond to two evolution paths in the Hilbert space, one correlated with the adiabatic displacement of the quantized field and the other correlated with free evolution. The Berry phase of the quantized field is manifested in the interference of the atomic qubit, other than in the interference of the field itself. As far as we know, our proposal is the first realistic one for directly measuring the Berry's adiabatic geometric phase for infinite-dimensional field states. When multiple qubits are involved, the acquired Berry phase leads to one-step implementation of important geometric quantum gates, which can not be achieved by previous methods. The effects of dissipation and the fluctuations in the parameters of the pump field on this phase and the atomic coherence are analyzed. The required qubit-boson coupling can be realized in cavity or circuit QED systems with atomic or superconducting qubits coupled

to a resonator.

The paper is organized as follows. In Sec.2 we propose a scheme for measuring the Berry phase of a quantized field using the Ramsey interference of an atomic qubit coupled to the quantized field and driven by an external field. We show that the acquired Berry phase can be used for one-step implementation of n -qubit quantum phase gates in Sec.3. In Sec.4 we investigate the geometric phase and visibility of the Ramsey interference fringes with the field decay and atomic spontaneous emission being included. Sec. 5 sees an analysis of the effect of the fluctuations in the parameters of the pump field on the Berry phase and atomic coherence. Finally we discuss the physical implementation of the model in microwave cavity QED and generalization of the ideas to the ion trap system, and we present a summary of our results in Sec. 6.

II. MEASUREMENT OF THE BERRY PHASE OF THE QUANTIZED FIELD

We first consider a two-level atom resonantly interacting with a single-mode quantized electromagnetic field and driven by a classical field. The dynamics of the whole system is described by the driven Jaynes-Cummings model [21]. We will label the upper and lower states of the two-level atom as $|e\rangle$ and $|g\rangle$ and describe its dynamics in terms of the Pauli operators σ_z , $\sigma^\pm = (\sigma_x \pm i\sigma_y)/2$. Then the Hamiltonian, in the interaction picture, is (assuming $\hbar = 1$)

$$H = \lambda(a^\dagger \sigma^- + a \sigma^+) + \Omega(\sigma^- e^{-i\phi} + \sigma^+ e^{i\phi}), \quad (1)$$

where a^\dagger and a are the creation and annihilation operators for the quantized field, λ is the atom-cavity coupling strength, and Ω and ϕ are the Rabi frequency and phase of the pump field. The first part of the Hamiltonian, describing the coherent energy exchange between the two-level atom and the quantized field, corresponds to the normal Jaynes-Cummings model [22]. The second part represents the coupling between the atomic transition $|g\rangle \leftrightarrow |e\rangle$ and the classical field. The eigenenergies of the driven Jaynes-Cummings model are the same as those of the normal Jaynes-Cummings model, while the field parts of the eigenstates are displaced in phase space with the displacement parameter determined by the Rabi frequency and phase of the pump field [21]. This allows one to adiabatically drive the quantized field to undergo a cyclic displacement evolution by slowly changing the parameters of the pump field. The Berry phase gained this way is determined by the area enclosed by the phase-space displacement trajectory, which is independent of the state of the system. To illustrate the idea clearly, we first consider the evolution of the dark state (the eigenstate with zero eigenenergy) of the Hamiltonian H . Such a state is $|\psi_0(\alpha)\rangle = |g\rangle |\alpha\rangle$, where $|\alpha\rangle$ is the coherent state of the quantized field with the parameter $\alpha = -\Omega e^{i\phi}/\lambda$. Adiabatic following of the dark state renders the phase

of the quantized field to be opposite to that of the pump field. When the parameters of the Hamiltonian complete a cyclic evolution after the duration T , the dark state describes a displacement loop of the quantized field and acquires a Berry phase

$$\begin{aligned} \beta &= i \int_0^T dt \langle \psi_0(\alpha) | \frac{d}{dt} | \psi_0(\alpha) \rangle \\ &= \frac{i}{2} \oint (\alpha^* d\alpha - \alpha d\alpha^*) = \pm 2A, \end{aligned} \quad (2)$$

where A is the area enclosed by the phase-space loop. The \pm sign depends on whether the sense of rotation is clockwise or counter-clockwise. Suppose that the Rabi frequency Ω of the pump field is kept constant and the phase ϕ , serving as the control parameter, is slowly varied from 0 to 2π . Then the state of the quantized field is moved around a circle with radius $|\alpha|$ in phase space and the acquired Berry phase is given by $-2\pi |\alpha|^2$. During the adiabatic evolution, the atom remains in the lower state $|g\rangle$ and the Berry phase completely originates from the cyclic quantum-mechanical displacement of the quantized field induced by the adiabatic variation of the Hamiltonian. This phase is significantly different from the nonadiabatic geometric phase produced by a coherent displacement force [23] in that the latter involves a nonzero dynamical component that is proportional to the total phase and thus cannot be removed [8, 24]. We note that the displacement evolution path of the quantized field is not affected if a term proportional to σ_z is added to the Hamiltonian. This implies that the acquired Berry phase is immune from the fluctuation in the qubit transition frequency as compared with the geometric phase of a qubit coupled only to a classical field.

In order to interpret the acquired Berry phase in the parameter space we rewrite the Hamiltonian as $H = \lambda[(a^\dagger \sigma^- + a \sigma^+) + \mathbf{B} \cdot \boldsymbol{\sigma}]$, where $\boldsymbol{\sigma} = \{\sigma_x, \sigma_y, \sigma_z\}$ and $\mathbf{B} = (\Omega \cos \phi/\lambda, \Omega \sin \phi/\lambda, 0)$ is the dimensionless effective vector field which acts as the control parameter. By cyclically changing Ω and/or ϕ the Hamiltonian describes a closed path in the two-dimensional parameter space $\{\mathbf{B}\}$. When the system is initially in the dark state of the Hamiltonian, the state of the system will follow the effective field and after a closed cycle it gains a purely geometric phase proportional to the area enclosed by the circuit traversed by the effective field. Note that, although the two-level system is coupled to the slowly changing effective vector field, its state is not varied when the system is initially in the dark state because the transition paths induced by the two fields with opposite phases interfere destructively. It is the quantized field whose state adiabatically follows the effective field \mathbf{B} and eventually acquires the Berry phase which depends upon the global property of the evolution path in the parameter space of the effective field.

The eigenstates of the Hamiltonian H with nonzero eigenvalues are $|\psi_{n+1,\pm}(\alpha)\rangle = D(\alpha)(|g\rangle |n+1\rangle \pm |e\rangle |n\rangle)/\sqrt{2}$, where $D(\alpha) = e^{\alpha a^\dagger - \alpha^* a}$ is the displacement

operator, and $n + 1$ is the quantum number of the displaced field state. After a complete evolution cycle all the eigenstates gain the same Berry phase β , which makes it impossible to observe the Berry phase in an interference experiment by initially preparing a superposition of different eigenstates as no relative geometric phase can be obtained. Neither can this phase be observed in optical interference experiments because it does not depend on the state of the quantized field. In the following, we show such a phase can be manifested in the interference between the probability amplitudes associated with the atomic state $|g\rangle$ and an auxiliary state $|f\rangle$ which is neither coupled to the pump field nor to the quantized field mode. The atom is initially driven to the superposition state $\frac{1}{\sqrt{2}}(|g\rangle + |f\rangle)$ from $|f\rangle$ using a classical pulse, which is analogous to the splitting of the photon beam at the first beam splitter of a Mach-Zehnder interferometer. The quantized field, initially prepared in the coherent state $|\Omega/\lambda\rangle$, acts as the dephasing element in one arm of the interferometer. After adiabatically dragging the Hamiltonian of Eq. (1) along a closed loop the quantized field undergoes a conditional cyclic displacement in phase space and introduces a relative phase between the two atomic states $|g\rangle$ and $|f\rangle$, leading to the superposition state $\frac{1}{\sqrt{2}}(e^{i\beta}|g\rangle + |f\rangle)$. This implies that the Berry phase acquired by the quantized field is encoded in the probability amplitude for finding the atom in the state $|g\rangle$. The interference between the probability amplitudes of the two superposed atomic states can be achieved through the transformations $|g\rangle \rightarrow \frac{1}{\sqrt{2}}(|g\rangle - |f\rangle)$ and $|f\rangle \rightarrow \frac{1}{\sqrt{2}}(|f\rangle + |g\rangle)$, which are analogous to the recombination of the photon beams at the second beam splitter of the Mach-Zehnder interferometer. Finally, the atomic state is measured. The probabilities of finding the atom in the states $|g\rangle$ and $|f\rangle$ are given by

$$P_{g,f} = \frac{1}{2}(1 \pm \cos \beta), \quad (3)$$

which is independent of the atom-field interaction time. By varying the Berry phase β the probability of finding the atom in a definite state exhibits Ramsey interference fringes. Therefore, the Berry phase of light field is manifested in the atomic Ramsey interference. With the choice $\Omega = \lambda/\sqrt{2}$ and $|\alpha| = 1/\sqrt{2}$, an adiabatic and cyclic evolution from $\phi = 0$ to $\phi = 2\pi$ achieves the Berry phase $\beta = -\pi$. Due to the presence of the Berry phase the atom is finally in the state $|g\rangle$. On the other hand, if no Berry phase is present, the atom is finally in the state $|f\rangle$. Therefore, the detection of the final state of the atom unambiguously distinguishes whether the Berry phase is present or not. It is important to note that even if the quantized field is in a macroscopic coherent state with $|\alpha| \gg 1$ the adiabatic and cyclic evolution of this field can be engineered by changing the Hamiltonian around a suitable circuit in parameter space, enabling the nonclassical Berry phase of light to be tested at the macroscopic level.

A modification of the interferometer can be applied to measure the Berry phase for any initial field state, for example, a thermal state. If the system is not initially in the dark state a purely geometric phase can be observed by applying a phase kick $-\sigma_z$ to the atom at time $T/2$. The product of the atomic state $|g\rangle$ with any displaced number state $D(\alpha)|n+1\rangle$ can be expressed as a superposition of the two eigenstates $|\psi_{n+1,+}(\alpha)\rangle$ and $|\psi_{n+1,-}(\alpha)\rangle$. During the adiabatic evolution these two eigenstates acquire opposite dynamical phases. The phase kick inverts these two eigenstates, effectively inverting the dynamical phases accumulated from time 0 to $T/2$. At time T the system completes a nontrivial cyclic evolution and the dynamical phase is completely canceled. The whole procedure results in a purely geometric phase β , which is determined by the area of the circuit followed by the effective field \mathbf{B} in parameter space and independent of the initial field state. This implies that the quantized field does not need to be prepared in a specific state since any field state can be expressed in terms of coherent state $|\alpha\rangle$ and displaced number states $D(\alpha)|n+1\rangle$. After the cyclic evolution all components acquire the same geometric phase β and the dynamical phases associated with all displaced number states $D(\alpha)|n+1\rangle$ are removed via the application of the atomic phase kick. The atomic interferometer can also be used to measure the geometric phase for the quantized field undergoing a noncyclic evolution. In this case the geometric phase can be expressed as the minus double of the area enclosed by the displacement trajectory and the straight line (the null phase curve) connecting the starting and ending points in phase space [25]. The geometric phase is directly related to the shift of the Ramsey interference fringes [26].

III. ONE-STEP IMPLEMENTATION OF MULTI-QUBIT PHASE GATES

Besides being of fundamental interest, geometric phases can be applied to design of quantum logic gates that have an intrinsic resilience against noise perturbations. We note that the conditional phase gates for n atomic qubits can be produced via a single conditional adiabatic displacement of the quantized field. The computational basis for each qubit is formed by the two states $|g\rangle$ and $|f\rangle$. If the transition $|g\rangle \leftrightarrow |e\rangle$ of each qubit is resonantly coupled to a quantized field and driven by a pump field, the interaction Hamiltonian is

$$H_n = \sum_{j=1}^n [\lambda_j(a^\dagger \sigma_j^- + a \sigma_j^+) + \Omega_j(e^{-i\phi} \sigma_j^- + e^{i\phi} \sigma_j^+)], \quad (4)$$

where the subscript j labels the qubits. Under the condition $\Omega_1/\lambda_1 = \Omega_2/\lambda_2 = \dots = \Omega_n/\lambda_n = r$ the Hamiltonian H_n has dark states of the form $|\phi_a\rangle |-re^{i\phi}\rangle$, where $|\phi_a\rangle$ can be any computational basis state except $|f_1 f_2 \dots f_n\rangle$. If the system is initially in the state $|\phi_a\rangle |-r\rangle$, slow variation of the phase ϕ from 0 to 2π produces a Berry phase

β . On the other hand, the state $|f_1 f_2 \dots f_n\rangle | -r \rangle$ is completely decoupled from the Hamiltonian H_n . As the state $|f\rangle$ is not coupled to the fields, transitions between degenerate dark states do not occur. Then the evolution of the qubit system proceeds as $|\phi_a\rangle \rightarrow e^{i\beta} |\phi_a\rangle$ and $|f_1 f_2 \dots f_n\rangle \rightarrow e^{i\beta} e^{-i\beta} |f_1 f_2 \dots f_n\rangle$. Discarding the trivial common phase factor $e^{i\beta}$, this is equivalent to an n -qubit controlled phase gate

$$U_n = e^{-i\beta |f_1 f_2 \dots f_n\rangle \langle f_1 f_2 \dots f_n|}, \quad (5)$$

in which if and only if all the qubits are in the state $|f\rangle$ the system undergoes a phase shift $-\beta$. This gate is essential to implementation of Grover's algorithms [27] and quantum Fourier transform [28]. Though any quantum computational network can be decomposed into a series of two- plus one-qubit logic gates, it may be extremely complex to construct an n -qubit phase gate using these elementary gates as the number of required operations increases exponentially with n . Thus, direct realization of n -qubit phase gates would greatly simplify practical implementation of certain quantum computational tasks. We note that the nonadiabatic geometric means [23] can be directly used for implementation of the n -qubit entangling gate $U_n' = \exp(i\theta J_z^2)$ where $J_z = \frac{1}{2} \sum_{j=1}^n (|f_j\rangle \langle f_j| - |g_j\rangle \langle g_j|)$, but it does not allow one-step implementation of the phase gate U_n .

Another important feature of the present gate operation is that it does not require the qubit-resonator coupling strengths λ_j to be identical. Furthermore, the conditional phase shift is not affected even if the transition frequencies of the qubits are different because the evolution of the quantized field is not changed when we add to the Hamiltonian H_n the terms $\sum_{j=1}^n \delta_j \sigma_{z,j}$, where δ_j is detuning between the transition frequency of qubit j and the field frequency. The tolerance to the nonuniformity of the qubits is important for the solid-state implementation of quantum computation since the parameters of artificial atomic qubits are usually not uniform. The geometric phase gate can also be implemented with the quantized field being initially in any state by applying the phase kick to all the atoms at time $T/2$.

IV. THE EFFECT OF DISSIPATION

Now let us derive the geometric phase and the visibility of the Ramsey interference fringes with the dissipation being included. The evolution of the system is governed by the master equation

$$\dot{\rho} = -i[H, \rho] + \mathcal{L}_\gamma \rho + \mathcal{L}_\kappa \rho, \quad (6)$$

where

$$\begin{aligned} \mathcal{L}_\gamma \rho &= \frac{\gamma}{2} (2a\rho a^\dagger - \rho a^\dagger a - a^\dagger a \rho), \\ \mathcal{L}_\kappa \rho &= \sum_j \sum_k \frac{\kappa_{j,k}}{2} (2S_{j,k}^- \rho S_{j,k}^+ - \rho S_{j,k}^+ S_{j,k}^- - S_{j,k}^+ S_{j,k}^- \rho), \end{aligned} \quad (7)$$

$S_{j,k}^+ = |j\rangle \langle k|$, $S^- = |k\rangle \langle j|$, γ is the decay rate for the quantized field, and $\kappa_{j,k}$ is the rate for the atomic spontaneous emission $|j\rangle \rightarrow |k\rangle$ ($|k\rangle$ being the levels lower than $|j\rangle$). The evolution of the system during the infinitesimal interval $[t, t + dt]$ is [29]

$$\rho(t) \rightarrow \rho(t + dt) = e^{\mathcal{L}_\kappa dt} e^{\mathcal{L}_\gamma dt} \mathcal{U}(t, dt) \rho(t), \quad (8)$$

where

$$\mathcal{U}(t, dt) \rho(t) = U(t, dt) \rho(t) U^\dagger(t, dt), \quad (9)$$

with $U(t, dt)$ being the unitary evolution operator governed by the slowly changing Hamiltonian H during $[t, t + dt]$. In the coherent state basis the action of the superoperator $e^{\mathcal{L}_\gamma dt}$ on the elements of the density matrix is given by [30]

$$e^{\mathcal{L}_\gamma dt} |\alpha_1\rangle \langle \alpha_2| = \langle \alpha_2 | \alpha_1 \rangle^{\gamma dt} |\alpha_1 e^{-\gamma dt}\rangle \langle \alpha_2 e^{-\gamma dt}|. \quad (10)$$

We here have discarded the atomic part in the density matrix element, which is not affected by $e^{\mathcal{L}_\gamma dt}$. The action of the superoperator $e^{\mathcal{L}_\kappa dt}$ is given by

$$\begin{aligned} e^{\mathcal{L}_\kappa dt} |g\rangle \langle g| &= e^{-\kappa_g dt} |g\rangle \langle g| + \sum_j \frac{\kappa_{g,j}}{\kappa_g} (1 - e^{-\kappa_g dt}) |j\rangle \langle j|, \\ e^{\mathcal{L}_\kappa dt} |f\rangle \langle f| &= e^{-\kappa_f dt} |f\rangle \langle f| + \sum_k \frac{\kappa_{f,k}}{\kappa_f} (1 - e^{-\kappa_f dt}) |k\rangle \langle k|, \\ e^{\mathcal{L}_\kappa dt} |g\rangle \langle f| &= e^{-(\kappa_g + \kappa_f)dt/2} |g\rangle \langle f|, \\ e^{\mathcal{L}_\kappa dt} |f\rangle \langle g| &= e^{-(\kappa_g + \kappa_f)dt/2} |f\rangle \langle g|, \end{aligned} \quad (11)$$

where $\kappa_g = \sum_j \kappa_{g,j}$, $\kappa_f = \sum_k \kappa_{f,k}$, and $|j\rangle$ and $|k\rangle$ are levels lower than $|g\rangle$ and $|f\rangle$, respectively. The dissipation makes the system deviate from the dark state of the Hamiltonian and acquire a dynamical phase, which can be removed by frequently performing the atomic phase kick $-\sigma_z$ during the course of the evolution of the system (M times with $M \gg 1$). When $M/T \gg \gamma$, κ_e , and κ_g , the dynamical effect is canceled before dissipation can affect it, where $\kappa_e = \sum_j \kappa_{e,j}$. When the dynamical effect is removed the action of the unitary evolution operator on the off-diagonal matrix elements is

$$\begin{aligned} &U(t, dt) |g\rangle \langle f| \otimes |\alpha_1\rangle \langle \alpha_2| U^\dagger(t, dt) \\ &= e^{-i\text{Im}(\alpha_1^* d\alpha_0)} |g\rangle \langle f| \otimes |\alpha_1 + d\alpha_0\rangle \langle \alpha_2|, \end{aligned} \quad (12)$$

where $\alpha_0 = -\Omega e^{i\phi}/\lambda$.

Suppose that the Rabi frequency Ω of the pump field is kept constant and the phase ϕ is slowly varied from 0 to 2π with the constant angular velocity $\omega = 2\pi/T$ during the interval $[0, T]$. For the initial state $\frac{1}{\sqrt{2}}(|g\rangle + |f\rangle)|-\Omega/\lambda\rangle$, the state of the system at time T is

$$\begin{aligned} \rho(T) &= \frac{1}{2} (e^{-\kappa_g T} |g\rangle \langle g| \otimes |\alpha_g\rangle \langle \alpha_g| + e^{-\kappa_f T} |f\rangle \langle f| \\ &\quad \otimes |\alpha_f\rangle \langle \alpha_f| + e^{-\Gamma + i\theta} |g\rangle \langle f| \otimes |\alpha_g\rangle \langle \alpha_f| \\ &\quad + e^{-\Gamma - i\theta} |f\rangle \langle g| \otimes |\alpha_f\rangle \langle \alpha_g|) \end{aligned}$$

$$\begin{aligned}
& + \frac{1}{2} \sum_j |j\rangle \langle j| \otimes \int_0^T \kappa_{g,j} e^{-\kappa_g t} |\alpha'_g\rangle \langle \alpha'_g| dt \\
& + \frac{1}{2} \sum_k \frac{\kappa_{f,k}}{\kappa_f} (1 - e^{-\kappa_f T}) |k\rangle \langle k| \otimes |\alpha_f\rangle \langle \alpha_f|,
\end{aligned} \tag{13}$$

where

$$\begin{aligned}
\alpha_g &= \frac{r}{\gamma + i\omega} (i\omega + \gamma e^{-\gamma T}), \\
\alpha'_g &= \frac{r}{\gamma + i\omega} (i\omega e^{i\omega T} + \gamma e^{-\gamma T}) e^{-(T-t)}, \\
\alpha_f &= r e^{-\gamma T}, \\
\Gamma &= (\kappa_g + \kappa_f)T/2 + \frac{r^2 \omega^2}{\gamma^2 + \omega^2} \left[\frac{1}{2} \gamma T + \frac{1}{4} (1 - e^{-2\gamma T}) \right], \\
\theta &= -r^2 \left[2\pi - \frac{\omega \gamma^2 T}{\gamma^2 + \omega^2} - \frac{\omega \gamma (\omega^2 - 2\gamma^2)}{(\gamma^2 + \omega^2)^2} (1 - e^{-2\gamma T}) \right],
\end{aligned} \tag{14}$$

$r = \Omega/\lambda$. We here have discarded the events that the atom spontaneously emits two or more photons. After the transformations $|g\rangle \rightarrow \frac{1}{\sqrt{2}}(|g\rangle - |f\rangle)$ and $|f\rangle \rightarrow \frac{1}{\sqrt{2}}(|f\rangle + |g\rangle)$, the probabilities of finding the atom in the states $|g\rangle$ and $|f\rangle$ are given by

$$P_{g,f} = \frac{1}{4} [e^{-\kappa_g T} + e^{-\kappa_f T} + \frac{\kappa_{g,f}}{\kappa_g} (1 - e^{-\kappa_g T}) \pm 2\nu \cos \beta], \tag{15}$$

where

$$\begin{aligned}
\nu &= \exp \left[-\Gamma + \frac{r^2 \omega^2}{\gamma^2 + \omega^2} (e^{-\gamma T} - \frac{1}{2} e^{-2\gamma T} - \frac{1}{2}) \right], \\
\beta &= \theta - \frac{r^2 \omega \gamma}{\gamma^2 + \omega^2} e^{-\gamma T}.
\end{aligned} \tag{16}$$

We here have assumed that the level $|f\rangle$ is lower than $|g\rangle$. Under the condition $\kappa_g, \kappa_f, \gamma \ll 1/T$, to first order correction the visibility of the interference fringes and geometric phase are approximately $\nu \simeq 1 - r^2 \gamma T - \kappa_{g,f} T/2$ and $\beta \simeq -|\alpha_0|^2 (2\pi + \gamma/\omega)$. The spontaneous emissions $|g\rangle \rightarrow |j\rangle$ ($j \neq f$) and $|f\rangle \rightarrow |k\rangle$ decrease the probabilities of finding the atom in the states $|g\rangle$ and $|f\rangle$, but do not affect the visibility of the interference fringes since the sum of the diagonal matrix elements and the sum of the off-diagonal matrix elements of the reduced density operator of the atom are shrunk by the same factor due to these spontaneous emissions. It should be noted that the second order spontaneous emissions $|j\rangle \rightarrow |k\rangle$ following $|g\rangle \rightarrow |j\rangle$ ($j \neq f$) or $|f\rangle \rightarrow |j\rangle$ affect neither the probabilities of finding the atom in the states $|g\rangle$ and $|f\rangle$ nor the visibility of the interference fringes. Due to the second order spontaneous emission $|f\rangle \rightarrow |k\rangle$ following $|g\rangle \rightarrow |f\rangle$ the correction to the visibility of the interference fringes is on the order of $\kappa_{g,f} \kappa_f T^2$.

V. THE EFFECT OF FLUCTUATIONS IN THE PARAMETERS OF THE PUMP FIELD

We now analyze the effect of the fluctuations in the parameters of the pump field on the Berry phase and atomic coherence. Set the fluctuations of the Rabi frequency and phase of the pump field to be $\delta\Omega(t)$ and $\delta\phi(t)$. Due to the presence of the fluctuation noise, the dark state of the system is $|g\rangle |\alpha(t)\rangle$, where $\alpha(t) = -[r_0(t) + \delta r(t)] e^{i\phi(t)}$, $\phi(t) = \phi_0(t) + \delta\phi(t)$, $r_0(t) = \Omega_0(t)/\lambda$, and $\delta r(t) = \delta\Omega(t)/\lambda$. Here $\Omega_0(t)$ and $\phi_0(t)$ are the Rabi frequency and phase of the pump field in the absence of noise. If the system is initially in the dark state $|g\rangle |\alpha(0)\rangle$ and the adiabatic condition is satisfied the final state is $e^{i\theta(T)} |g\rangle |\alpha(T)\rangle$, where $\theta(T) = -\int_{\phi(0)}^{\phi(T)} (r_0 + \delta r)^2 d\phi$. To the first order correction the geometric phase is

$$\beta \simeq \beta_0 - \int_0^T 2r_0 \delta r \dot{\phi}_0 dt - \int_0^T r_0^2 \delta \dot{\phi} dt + r_0^2(0) \sin \delta\phi(T), \tag{17}$$

where β_0 is the Berry phase in the absence of noise. We here have assumed $\delta r(0) = 0$ and $\delta\phi(0) = 0$. Without loss of the generality, we again consider the case that Ω_0 is kept constant and ϕ_0 undergoes an adiabatic cyclic evolution from 0 to 2π with the constant change rate $\dot{\phi}_0 = 2\pi/T$. Then the expression reduces to $\beta \simeq \beta_0 - \frac{4\pi}{T} r_0 \int_0^T \delta r dt$. We here assume that the fluctuations are Gaussian and Markovian processes with the bandwidth Γ_Ω (Γ_ϕ) and intensity σ_Ω^2 (σ_ϕ^2) for $\delta\Omega$ ($\delta\phi$). Under the condition $\Gamma_\Omega, \Gamma_\phi \ll \lambda$, the pump field fluctuations are adiabatic with respect to the Rabi frequency of the dressed atom-field system [7]. Consequently, the geometric phase is Gaussian distributed with the mean value equal to the noiseless Berry phase. Its variance is given by

$$\sigma_\beta^2 = 16\pi |\beta_0| \frac{\sigma_\Omega^2}{(\lambda \Gamma_\Omega T)^2} (\Gamma_\Omega T - 1 + e^{-\Gamma_\Omega T}). \tag{18}$$

In the limit of the evolution being much slower than the fluctuation of the Rabi frequency ($\Gamma_\Omega T \gg 1$), the variance approximates $\sigma_\beta^2 = 16\pi |\beta_0| \sigma_\Omega^2 / (\lambda^2 \Gamma_\Omega T)$, which vanishes in the limit $(\Gamma_\Omega T)^{-1} \rightarrow 0$. When $\Gamma_\Omega T \ll 1$, the variance reduces to $\sigma_\beta^2 = 8\pi |\beta_0| \sigma_\Omega^2 / \lambda^2$. This implies that the geometric phase is sensitive to slow fluctuations, with the variance being path-dependent like the geometric phase itself, coinciding with the geometric dephasing for a spin 1/2 in a slowly changing magnetic field [7]. As long as the adiabatic condition is satisfied, the fluctuation does not induce any correction to the dynamical phase as the spectrum of the Hamiltonian does not depend on the parameters of the pump field. Thus the dephasing of this system is different from that of the spin 1/2 to which the main contribution has the dynamical origin [7].

Let us consider the effect of fluctuations in the classical control parameters on the atomic Ramsey interference. In the presence of the noise, after the conditional displacement evolution the system is in a mixed state,

whose density operator is given by

$$\rho = \int |\psi\rangle \langle\psi| P(\beta) P(\delta r(T)) P(\delta\phi(T)) d\beta d\delta r(T) d\delta\phi(T), \quad (19)$$

where

$$|\psi\rangle = \frac{1}{\sqrt{2}} [e^{i\theta(T)} |g\rangle |\alpha(T)\rangle + |f\rangle |\alpha(0)\rangle], \quad (20)$$

$P(\beta)$, $P(\delta r(T))$, and $P(\delta\phi(T))$ are Gaussian distribution functions for β , $\delta r(T)$, and $\delta\phi(T)$, respectively. The coherence between the two atomic states is shrunk by a factor $F = e^{-\sigma_\beta^2/2 - \sigma_\Omega^2/(2\lambda^2) - |\beta_0|\sigma_\phi^2/(4\pi)}$ due to random distributions in the Berry phase and the noncyclic evolution of the field state correlated with the atomic state $|g\rangle$.

Now we analyze how the classical noises affect the fidelity of the n -qubit quantum gates. Set the initial state of the qubit system to be $c_1 |\psi_a\rangle + c_2 |f_1 f_2 \dots f_n\rangle$, where $|c_1|^2 + |c_2|^2 = 1$ and $|\psi_a\rangle$ can be any normalized superposition of the computational basis states except $|f_1 f_2 \dots f_n\rangle$. Due to the fluctuation perturbation the density operator of the whole system again has the form of Eq. (19), where

$$|\psi\rangle = c_1 e^{i\theta(T)} |\psi_a\rangle |\alpha(T)\rangle + c_2 |f_1 f_2 \dots f_n\rangle |\alpha(0)\rangle. \quad (21)$$

The infidelity caused by the fluctuations is $\epsilon = |c_1 c_2|^2 (1 - F)$. The result is valid even if these qubits are entangled with other qubits not involved in the gate operation.

VI. DISCUSSION AND CONCLUSION

For potential physical implementation of the model, we consider microwave cavity QED experiments with circular Rydberg atoms and a superconducting millimeter-wave cavity, which has a remarkably long damping time $T_c = 0.13$ s [31]. The states $|f\rangle$, $|g\rangle$, and $|e\rangle$ are the circular states with principal quantum numbers 49, 50, and 51, respectively. The corresponding atomic radiative time is about $T_r = 3 \times 10^{-2}$ s. The transition $|g\rangle \longleftrightarrow |e\rangle$ is strongly coupled to the cavity mode with the coupling strength $\lambda = 2\pi \times 25$ kHz. The pump field can be provided by a classical microwave source. The adiabatic approximation is valid under the condition that the time scale T of the variation of the control parameter is longer than the inverse of the energy gap $\delta E = \lambda$ between the dark state and the nearest bright states with nonzero eigenenergies, i.e., the dynamical time scale. If we set $T = 20/\lambda$, then the leakage error to the excited eigenstates is on the order of $1/(\lambda T)^2 = 2.5 \times 10^{-3}$. Suppose that the Rabi frequency Ω of the pump field is kept constant and the phase ϕ is slowly varied from 0 to 2π . Then due to dissipation the visibility of interference fringes is approximately reduced by $(\Omega/\lambda)^2 T/T_c + \kappa_{g,f} T/2$, which is on the order of 10^{-3} . The correction to the geometric phase is $-2\pi(\Omega/\lambda)^2 T/T_c$, also on the order of 10^{-3} . Therefore, the adiabatic condition can be perfectly satisfied and the influence of decoherence is negligible. We

note that the atomic phase kick has been experimentally achieved by applying a fast electric field pulse [32], which allows the measurement of the geometric phase for the cavity field even if the system is not in the dark state. In experiment, it may be more convenient to keep the phase of the pump field unchanged, but detune its frequency from the frequency of the quantized field by an amount δ with $\delta \ll \lambda$. Then $t\delta$ takes the role of the slowly varying phase ϕ . An alternative physical system to implement the required Hamiltonian is the circuit QED setup, in which superconducting qubits are strongly coupled to the resonator field. In fact, the Hamiltonian of Eq. (1) has been experimentally realized using a phase qubit coupled to a superconducting coplanar waveguide resonator and driven by an external microwave pulse [33].

The above ideas can be directly applied to the ion trap system. Consider an ion trapped in a harmonic potential and driven by two laser beams tuned to the carrier and the first lower vibrational sideband with respect to the electronic transition $|g\rangle \rightarrow |e\rangle$ and the vibrational mode. In the Lamb-Dicke limit, the coupling between the internal and external degrees of freedom of the trapped ions is described by the Hamiltonian (1), with the Rabi frequency and phase of the laser for the carrier excitation serving as the control parameters. The Berry phase acquired by the vibrational mode after an adiabatic displacement evolution in phase space is directly manifested in the Ramsey interference between $|g\rangle$ and an auxiliary state $|f\rangle$ decoupled from the Hamiltonian. It is worthwhile to note that a scheme has been proposed for measuring the geometric phase in the vibrational mode of a trapped ion by adiabatically varying the squeezing parameter in the engineered squeezing operator [34]. However, the scheme requires squeezing transformations with opposite squeezing parameters before and after the adiabatic evolution, discrimination between two nonorthogonal coherent states, and techniques to cancel the accumulated dynamical phase.

We have shown how to produce and observe the non-classical Berry phase of an electromagnetic field in a generic qubit-boson system involving a qubit coupled to a quantized field and driven by a classical pump field. The adiabatic variation of the parameters of the pump field forces the quantized field mode to displace along a loop in phase space, producing a purely geometric phase. The origin of the geometric phase is the quantum nature of the field mode without any classical counterpart. When the system is initially in the dark state, the geometric phase can be directly detected using the atomic Ramsey interferometer. Otherwise, one should apply a phase kick to the atom to cancel dynamical contributions. Besides fundamental interest, geometric manipulation of the quantized field opens new possibilities for robust implementation of important quantum phase gates in a single step. The effects of both the quantum and classical noises on the Berry phase and visibility of the Ramsey interference fringes are analyzed.

Note added. Since completion of this work, two

preprints by Vacanti et al. [35] and Pechal et al. [36] investigating the measurement of the geometric phase of a quantum harmonic oscillator using the interference of a qubit have appeared. The first one proposed a scheme to measure the nonadiabatic geometric phase produced by a coherent displacement force. The second one reported an experiment for observing the Berry's adiabatic geometric phase in circuit QED. In the experiment, the acquired phase includes a dynamical contribution, and a purely geometric phase cannot be produced by the cyclic evolution of the Hamiltonian. The geometric phase is only indirectly measured by evaluating it as the differ-

ence between the phases for the circular path and the phase for a straight line in the phase space. In comparison, the present scheme allows direct measurement of this phase.

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